DE-Vol. 62, Vibration Isolation, Acoustics,
and Damping in Mechanical Systems
ASME 1993

A NUMERICAL APPROACH FOR THE PREDICTION AND REDUCTION OF STRUCTURE-BORNE NOISE DURING THE DESIGN STAGE OF GROUND VEHICLES

Nickolas Viahopoulos
Automated Analysis Corp.
Ann Arbor, Michigan

Edward V. Shalis
U.S. Army Tank Automotive Command
R&D Center, Countermeasures
Warren, Michigan

Michael A. Latcha
Oakland University
Rochester, Michigan

ABSTRACT

During the design stage of ground vehicles it is important to reduce the noise emitted from structural components. In commercial applications the reduction of the interior noise for passenger comfort is a concern with increased significance. In military applications noise radiated from the exterior of the vehicle is of primary importance for the survivability of the vehicle. Numerical acoustic prediction software can be used during the design stage to predict and reduce the radiated noise. Two formulations, the Rayleigh integral equation[1] and the direct boundary element method[2,3] were implemented into software for acoustic prediction. The developed code can accept information from a finite element model with a known input forcing function. Specifically, the predicted velocities on the structural surfaces can be used as input to the acoustic code for predicting the noise emitted from a vibrating structure. Computation of acoustic sensitivities[4] was also implemented in the code. This information can identify the portions of the boundary that effect the radiated noise most, and it can be used in an optimization process to reduce the noise radiated from a vibrating structure.

INTRODUCTION

A computer implementation of two formulations (Rayleigh integral, and direct boundary element method) for numerical noise prediction is presented in this paper. A method to compute the acoustic sensitivities in the boundary element method was also developed and implemented in the code. A numerical approach was developed to evaluate the boundary element integrals with 1/r singularities. This technique eliminates the necessity of mapping the singular elements to a polar coordinate system[6,7]. The overdetermination technique[8] is used to eliminate numerical instabilities in radiation problems. The resulting system of equations is transformed to a square system by the use of matrix algebra[2]. This technique simplified the implementation of acoustic sensitivities[4] in the developed code. The latter can be used to identify the portions of the boundary that effect the radiated noise most. This information can be used in an optimization process to reduce the noise radiated from a vibrating structure.

The mathematical and numerical implementation for the singular integration and the computation of the acoustic sensitivities are presented. These constitute new features in the development. The interpretation of the acoustic sensitivities is also explained. The numerical solution of the Rayleigh formulation is compared to the analytical solution of a vibrating piston, and the boundary element implementation is verified by comparison to the analytical solution of a pulsating sphere and test data. A typical application of the noise generated in the interior of a vehicle is presented along with the acoustic sensitivity results that can be used to improve the design.

The acoustic prediction code presented in this paper constitutes the first structural noise prediction sub-module that will be integrated into the much larger vehicle noise prediction model (RANES). This program predicts the external radiated field due to components making up ground vehicles. At the present time, airborne and structure-borne component sources such as engines, cooling fans, and transmissions form the core of the model. However, the structural paths are very basic and require more sophisticated tools for modeling. Initially the direct boundary element code and the Rayleigh code presented here will be integrated into RANES. In the future other alternative methods will be investigated and incorporated into the program[9].

NUMERICAL IMPLEMENTATION

The Rayleigh integral method is based on the fundamental solution of a point source placed on a rigid wall. The surface of the vibrating structure is considered to be comprised by many point sources placed in a baffle[10]. Then the acoustic response at any field point is computed as the integral of the fundamental solution over the structural surface.

$$p = \frac{i\,\omega\,\rho}{2\,\pi} \int_s \frac{e^{-ikr}}{r} \vec{v}\, d\vec{s} \tag{1}$$

where p = acoustic pressure at a field point, r = distance from the source to the point where the acoustic pressure is computed, ω = circular frequency, ρ = density of acoustic medium, $\vec{v}$ = surface velocity vector of the differential area, and $\vec{ds}$ = the differential area vector. The applicability of this method is limited in predicting the noise emitted from plane structures, but it has significant computational advantages, when it can be used, over the general boundary element method.

The Helmholtz integral equation[5, 6, 11, 12] constitutes the basis of the boundary element method. It is a general formulation that can provide solution to both exterior radiation and interior acoustics problems. A direct approach with primary acoustics variables $p(r_s)$ = acoustic pressure, and $\vec{v}(r_s)$ = acoustic velocity at point r_s of the boundary element model is employed. The acoustic pressure $p(r_o)$ at a point r_o is expressed as:

$$-i\,k\,\rho\,c \int_S \left(\Psi(r_s, r_o)\, \vec{v}(r_s)\, \hat{\eta}(r_s)\right) dS$$

$$-\int_S p(r_s) \frac{e^{-ik\left|\vec{r}_s - \vec{r}_o\right|}}{\left|\vec{r}_s - \vec{r}_o\right|} \left(\frac{1}{\left|\vec{r}_s - \vec{r}_o\right|} - i\,k\right) \frac{\hat{\eta}(r_s)\left(\vec{r}_s - \vec{r}_o\right)}{\left|\vec{r}_s - \vec{r}_o\right|} dS = C\,p(r_o) \tag{2}$$

where $\Psi(r_s, r_o)$ = Green's function, k = wave number, C = constant depending on the location of the observation point ro, and $\hat{\eta}(r_s)$ = unit normal at point r_s.

By descritizing equation (2) and performing the numerical integration in terms of the normal coordinates within an element results in:

$$\sum_{m=1}^{M} \Bigg[-i\,\omega\,r\,\hat{\eta}_m A_m \sum_{i=1}^{n} \left(\sum_{j=1}^{3} \frac{e^{-ik\left|\vec{r}_{smj} - \vec{r}_o\right|}}{\left|\vec{r}_{smj} - \vec{r}_o\right|} \vec{v}(r_{smj})\, \eta_j^i\right) w_i$$

$$- A_m \sum_{i=1}^{n} \Bigg(\sum_{j=1}^{3} p(r_{smj}) \frac{e^{-ik\left|\vec{r}_{smj} - \vec{r}_o\right|}}{\left|\vec{r}_{smj} - \vec{r}_o\right|} \left(\frac{1}{\left|\vec{r}_{smj} - \vec{r}_o\right|} - i\,k\right)$$

$$\frac{\hat{\eta}_m(\vec{r}_{smj} - \vec{r}_o)}{\left|\vec{r}_{smj} - \vec{r}_o\right|} \eta_j^i\Bigg) w_i\Bigg] = C\,p(r_o) \tag{3}$$

where m = element number, $\hat{\eta}_m$ = unit normal on the mth element, n = order of integration scheme, $\eta_j^i = j^{th}$ normal coordinate corresponding to the i^{th} integration point, and $\vec{r}_{smj}$ defines the location of the j^{th} node of the n^{th} element. On the vibrating structural surface S the velocities $\vec{v}(r_{smj})$ are the known boundary conditions from the structural finite element analysis. In order to be able to predict acoustic pressure at field point locations, the acoustic pressures on the vibrating surface must be computed first. Therefore, a two step process is required. First the observation point is positioned at every node of the discretized surface S. Using equation (3) one relation can be written for each surface node resulting in a system with as many equations as nodes. When the observation point is located at one of the nodes of an element the terms $\frac{1}{\left|\vec{r}_{smj} - \vec{r}_o\right|}$ become singular for that element when $\vec{r}_o = \vec{r}_{smj}$.

The second term on the left side of equation (3) does not present any computational problems since $\hat{\eta}_m(\vec{r}_{smj} - \vec{r}_o)$ is zero when $\vec{r}_o$ is located at a node of the m^{th} element, and the associated term becomes zero. The term

$$-i\,\omega\,r\,\hat{\eta}_m A_m \sum_{i=1}^{n} \left(\sum_{j=1}^{3} \frac{e^{-ik\left|\vec{r}_{smj} - \vec{r}_o\right|}}{\left|\vec{r}_{smj} - \vec{r}_o\right|} \vec{v}(r_{smj})\, \eta_j^i\right) w_i$$

is the one that presents the 1/r singularity when $\vec{r}_o = \vec{r}_{smj}$. In previous work this singularity is treated by performing a transformation to a local system of polar coordinates[6,11,12]. In this work the following numerical solution scheme is followed. The singularity occurs because the function $f = \frac{e^{-ik\left|\vec{r}_{sm}^{\,i} - \vec{r}_o\right|}}{\left|\vec{r}_{sm}^{\,i} - \vec{r}_o\right|} \vec{v}(r_{sm}^i)$ at the integration points is expressed as :

$$f = \sum_{j=1}^{3} \frac{e^{-ik\left|\vec{r}_{sm} - \vec{r}_o\right|}}{\left|\vec{r}_{sm} - \vec{r}_o\right|} \vec{v}(r_{smj})\, \eta_j^i \tag{4}$$

and when $\vec{r}_o = \vec{r}_{smj}$ function f becomes singular. In the integration scheme followed in this work, the integration points are positioned within the element. The value $\vec{r}_{sm}^{\,i}$ is computed first as $\vec{r}_{sm}^{\,i} = \sum_{j=1}^{3} \vec{r}_{smj}\eta_j^i$ and then

$$f = \frac{e^{-ik\left|\vec{r}_{sm}^{\,i} - \vec{r}_o\right|}}{\left|\vec{r}_{sm}^{\,i} - \vec{r}_o\right|} \sum_{j=1}^{3} \vec{v}(r_{smj})\, \eta_j^i \qquad (5)$$

In this manner the singularity is avoided numerically.

By using equations (3) and (5), and by positioning $\vec{r}_o$ at every node on the discretized surface S, a linear system of equations is assembled:

$$[A]\{p\} = \{d\} \qquad (6)$$

where the coefficients of matrix [A] are computed from the second term of the left side of equation (3) and the right side of the same equation, {d} is computed from the known surface velocities of the vibrating structure and the first term of the left side of equation (3), and {p} is the vector of unknown acoustic pressure on the surface S. Once {p} has been computed equation (3) can be used to compute the acoustic pressure at any point $\vec{r}_o$ in the acoustic space in the form of the vector equation

$$\{A\}^T\{p\} + \{B\}^T\{v\} = p(\vec{r}_o) \qquad (7)$$

where terms of the vector {A}, {B} are computed from the integrals of the left side of equation (3), {p} = vector of acoustic pressure on the surface S, and $p(\vec{r}_o)$ = acoustic pressure at point r_o.

For radiation problems the solution to equation (6) becomes unstable[13] and the overdetermination method is used to overcome this problem. Specifically, observation points can be specified in the interior of the surface S. In this manner additional equations with specified solution are added to the N x N system (6), (N = number of nodes), resulting in

$$\underset{M \times N}{[A]} \;\; \underset{N \times 1}{\{p\}} = \underset{N \times 1}{\{d\}} \qquad (8)$$

In order to get a square system of equations, both sides of equation (8) are pre - multiplied by $[A]^T$ resulting in a square complex system of equations with N unknowns[2]. This technique is simpler than the Lagrange multiplier method, and the resulting system of equations can be readily used in a formulation to compute acoustic sensitivities[4].

The numerical solution scheme presented in this section is implemented in computer code. The software can be used to solve radiation and interior acoustics problems.

Acoustic Sensitivities

The acoustic sensitivity with respect to a sizing design variable is denoted by $\frac{dp_o}{dh}$. The term sizing indicates that the change in the physical property h does not influence the shape and the geometry of the structure. The design variable h for example can be the thickness of a plate, the stiffness of a spring, the cross sectional area of a beam, a material property, etc.

By using the chain rule, $\frac{dp_o}{dh}$ can be written:

$$\frac{dp_o}{dh} = \sum_{i=1}^{N} \frac{dp_o}{du_{ni}} \frac{du_{ni}}{dh} \qquad (9)$$

where u_{ni} = normal velocity of the i^{th} node, and N = total number of nodes used in the model. In this manner the acoustic sensitivity is decomposed in two parts, $\frac{dp_o}{du_{ni}}$ which can be calculated from the BEM acoustic analysis, and $\frac{du_{ni}}{dh}$ which can be calculated by the FEM structural analysis. The first part indicates how the acoustic pressure changes with respect to the normal velocity boundary conditions (acoustic part). The second part indicates how the boundary conditions change with respect to the design variables (structural part). In this work the computation of the first part $(\frac{dp_o}{du_{ni}})$ is numerically implemented in the BEM acoustic prediction acoustic code. MSC/NASTRAN version 67 can be used for computation of structural dynamic sensitivities (SOL 108, SOL 111)[14].

In order to compute $(\frac{dp_o}{du_{ni}})$ a two step process, similar to the one used to calculate p_o, is used. Equation (6) is differentiated with respect to the i^{th} node normal velocity u_{ni} :

$$\frac{d[A]}{du_{n_i}}\{p\} + [A]\frac{d\{p\}}{du_{n_i}} + \frac{d[B]}{du_{n_i}}\{u_n\} + [B]\frac{d\{u_n\}}{du_{n_i}} = \{0\} \qquad (10)$$

In this equation $\frac{d[A]}{du_{n_i}}, \frac{d[B]}{du_{n_i}} = 0$, because [A] and [B] are only geometry dependent, and change of the velocity u_{ni} does not imply changes in the geometry, since only sizing variables are considered in this formulation. Therefore from Eq. (10):

$$\frac{d\{p\}}{du_{n_i}} = -[A]^{-1}[B]\{I_i\} \qquad (11)$$

where $\{I_i\}$ = unit vector with only one non - zero component in the i^{th} position. Differentiating equation (7) with respect to u_{n_i} and considering equation (11) results in:

$$\frac{dp_o}{du_{n_i}} = \left\{-\{A_o\}^T[A]^{-1}[B] + \{B_o\}^T\right\}\{I_i\} \qquad (12)$$

This constitutes the derivation of the acoustic part of the acoustic sensitivity $\frac{dp_o}{dh}$ (eq. 9).

The derivative of the acoustic pressure p at a field point, with respect to a variable c, can be written as:

$$\frac{dp}{dc} = \frac{d(p_r + i\,p_i)}{d(c_r + i\,c_i)} = \frac{dp_r + i\,dp_i}{dc_r + i\,dc_i} = A + i\,B \qquad (13)$$

where d = differentiation operator, p_r and p_i are the real and imaginary part of the acoustic pressure, c_r and c_i are the real and imaginary parts of the complex variable c, A and B are the real and imaginary parts of dp/dc. When the variations dc_r and

dc_i are known, equation (13) can be used to compute the corresponding values of dp_r and dp_i as:

$$dp_r = A\,dc_r - B\,dc_i$$
$$dp_i = B\,dc_r + A\,dc_i \qquad (14)$$

Based on the real and imaginary value of p, equation 14 can be used to determine how c_r and c_i must change to reduce sound pressure.

VERIFICATION

The analytical solution for a baffled piston vibrating with uniform velocity was used to verify the software developed for the Rayleigh method. The pressure along the perpendicular to the center of the piston was computed. A piston of 1 meter radius and 1m/sec uniform velocity was used in the verification. Results at 1350 Hz are presented in Figure 1. Analyses at 100Hz, 300Hz, 400Hz, 500Hz, and 1000Hz were also performed resulting in good correlation between numerical and analytical results.

The analytical solution of a pulsating sphere[2,6] was used to verify the numerical implementation of the Boundary Element Solution with the approach of treating numerically the 1/r singularities, and the implementation of the overdetermination technique for irregular frequencies. A sphere with a = 1m radius, and U = 1m/sec radial velocity was used in this application. Results were computed for three points located at 6m distance from the center of the sphere on each of the three principal axis. Results for all three points coincide as expected. The results were compared to the analytical solution. Figure 2 presents the results when using one overdetermination point within the sphere.

In order to determine whether the singular frequencies are important in complex geometry the noise radiated from a box was computed. The size of the box was 1m x 0.4m x 0.5m. From the analytical solution of the normal modes the first mode along the longest direction (x - axis) occurs at 570 Hz for zero pressure boundary condition. A uniform velocity was applied on the side vertical to x - axis. Analysis was performed between 550 Hz and 590Hz without any overdetermination point. Results for the acoustic response at an exterior field point are plotted for the frequency range 550Hz - 590Hz (Figure 3). As it can be seen, there is no significant variation in the results due to the singular frequency. This indicates that in practical applications where the shape of the radiating structure is complex and the velocity boundary conditions do not match exactly the surface velocity corresponding to the interior Dirichlet mode[13] the singular frequencies might not be important. Further investigation is needed in that area.

In addition to the analytical solution, comparison to test data was also performed for the boundary element code. A cube fixture was constructed for testing. The four lateral sides of the box were 1 inch thick, while the top flexible side was 0.031 inch thick. An acoustic boundary element model was created to perform the acoustic analysis. Results from the test and the analysis were retrieved at a field point plane 0.2 meters above the flexible upper side of the box. The test and the analysis were performed at frequencies 77 Hz, 131 Hz, and 160 Hz. Numerical and test data for all the field points at 77Hz are presented in Figure 4. The average numerical and test intensity values for all the frequencies are summarized in Table 1.

Frequency (Hz)	Numerical (dB)	Test (dB)
77	99.4	99.8
131	95.6	94.1
160	88.3	86.5

Table 1. Summary of Average Computed and Measured Intensities

APPLICATION

An application case was developed to demonstrate the potential use of the developed acoustic code. The software can interface with NASTRAN to input the Boundary Element mesh, the postprocessing surfaces, and the results of a structural forced response finite element analysis. The postprocessing surfaces are used for visualizing the results. They do not effect the analysis in any manner. They function as an array of microphones measuring noise at the node locations of the post - processing surface. The output acoustic pressures on the nodes of the postprocessing surfaces are provided in PDA/PATRAN format.

Acoustic analysis for the interior of a car compartment was performed as a typical application of interior acoustics. A generic car interior model was used in this analysis. The main dimensions of the compartment were 2.4m length, 1.4m width, 1.0m height. Two field point surfaces were defined in the interior to postprocess the results. In this application, only noise radiated from the roof panel was considered. Analysis was performed at 150Hz. The results for the acoustic pressure in dB values are shown in Figure 5. Results for the acoustic sensitivities with respect to the interior point (1.39, 0, 0.88) are presented in Figures 6.a and 6.b (real and imaginary part of $\frac{dp_o}{d\dot{u}_{ni}}$ respectively). This information can be used to identify the structural modifications that reduce the radiated noise.

SUMMARY

A numerical implementation of two methods for exterior and interior noise analysis are presented in this paper. The development, implementation, and interpretation of acoustic sensitivities is also presented. A numerical technique was used to evaluate the boundary element integrals with 1/r singularities. Acoustic sensitivity computations are also part of the development. In combination with the structural dynamic sensitivities computed by MSC/NASTRAN they can provide information about the sizing design variables that lead to noise reduction. Closed form analytical solutions are used to verify the development and the overdetermination technique used in the code. Test data are also used for comparison to numerical results and establish further confidence in the development. Preliminary investigation shows that the numerical singularity at the natural frequencies of the interior Dirichlet modes might not be important in practical applications with complex geometry and vibration velocities that do not match the velocity distribution of the interior Dirichlet modes.

The scope of this development is to provide a customized acoustic boundary element software that can be used with military acoustic signature prediction codes. The recent emphasis by the military on predicting the acoustic signature of its design vehicles has initiated a need for acoustic models that are based on automotive component noise data and analytical models. Knowledge of the noise radiated by the components under parametric field operations would then translate to requirements on the individual components as well as predicting the ground vehicle noise signature before the development begins. TACOM is developing the Radiated Noise Estimation Software (RANES)[15] that addresses both airborne and structure-borne sources of noise for several military wheeled and tracked vehicles. The airborne sources have been partially defined during the first phase of development. The acoustic prediction code presented here constitutes the first structural noise prediction sub-modules of the development. In addition, acoustic detection range prediction models can predict the sound propagation at long distances (ADRPM-V, ADRPM-VII)[16]. By taking into consideration the effect of the ground, the atmospheric conditions (absorption, refraction due to temperature gradients, and wind), the size of the target, any barriers between the target and the detector, and the distance between them, it can predict the acoustic signature of the vehicle. Information about noise in the near field must be input into the program in the form of one - third octave band distribution. This information can be provided by the developed boundary element code.

ACKNOWLEDGMENTS

This work was sponsored by the U.S. Tank Automotive Command under contract number DAAE07-92-C-R028.

REFERENCES

1. L.E. Kinsler, A.R. Frey, A.B. Coppens, and J.V. Sanders, *Fundamentals of Acoustics*, Third Edition, John Wiley & Sons, New York, 1982.

2. G.H. Koopman and H. Benner, "Method for computing the sound power of machines based on the Helmholtz Integral," J. Acoust. Soc. Am, Vol. 71, pp. 78-89, (1982).

3. A.F. Seybert, B. Soenarko, F.J. Rizzo,and D.J. Shippy, "Application of the BIE Method to Sound Radiation Problems Using an Isoparametric Element", Journal of Vibration, Acoustics, Stress and Reliability in Design, Vol. 106, pp. 414-421, (1984).

4. N. Vlahopoulos, "Boundary Element Formulations for Acoustic Sensitivities with Respect to Structural Design Variables, and Acoustic Impedance", Second International Congress or Recent Developments in Air - and Structure - Borne Sound and Vibration, Auburn University, March 1992.

5. R.D. Ciskowski, and C. A. Brebia, "Boundary Element Methods in Acoustics," Computational Mechanics Publications, 1991.

6. A.F. Seybert, B. Soenarko, F.J. Rizzo, and D.J. Shippy, "Radiation and Scattering of Acoustic Waves in Three Dimensions", J. Acoust. Soc. Am, Vol. 77, pp. 362-368, (1985).

7. F.J. Rizzo, and D.J. Shippy, "An Advanced Boundary Integral Equation Method for Three - Dimensional Thermoelasticity", International Journal for Numerical Methods in Engineering, Vol. 11, pp. 1753-1768, (1977).

8. H. Scherck, "Improved Integral Formulation for Acoustic Radiation", J. Acoust. Soc. Am, Vol. 44, pp. 41-58, (1968).

9. J.F. Wilby, and L.D. Pope, "Band-Limited Power Flow into Enclosures," JASA, Vol 62, No. 4, pp. 906 - 910, October 1977.

10. L. Cremer, M. Heckl, and E. E. Ungar, "Structure - Borne Sound," Springler - Verlag, 1973.

11. R.J. Bernhard, B.K. Gardner, C.B. Mollo, and C.R. Kipp, "Prediction of Sound Fields in Cavities Using Boundary Element Methods", AIAA Journal, Vol. 25, No. 9, pp. 1176-1183 (1986).

12. K.R. Fyfe, and J.P. Coyette, "Acoustical Design Using the Boundary Element Method", Second International Conference on Methodology and Innovations in Automotive Experimentation, Florence, Italy, November 1988.

13. S. R. Hahn, A. A. Ferri, J. H. Ginsberg, "A Review and Evaluation of Methods to Alleviate Non-Uniqueness in the Surface Helmholtz Integral Equation," Structural Acoustics, NCA-Vol.12/AMD-Vol.128, ASME 1991.

14. *MSC/NASTRAN User's Manual, Version 66*, The MacNeal - Schwendler Corporation, 1989.

15. "Initial Development of RANES Acoustic Signature Estimation and Analysis Software System for Military Ground Vehicles," Contract number DAAE07-89-C-R015, U.S. Army Tank-Automotive Command Research, Development, and Engineering Center.

16. E. Shalis, and G. Koenig, "ADRPM-VII Applied to the Long-Range Acoustic Detection Problem," Fourth International Symposium on Long - Range Sound Propagation, Langley Research Center, Hampton, Virginia, May 16-17, 1990.

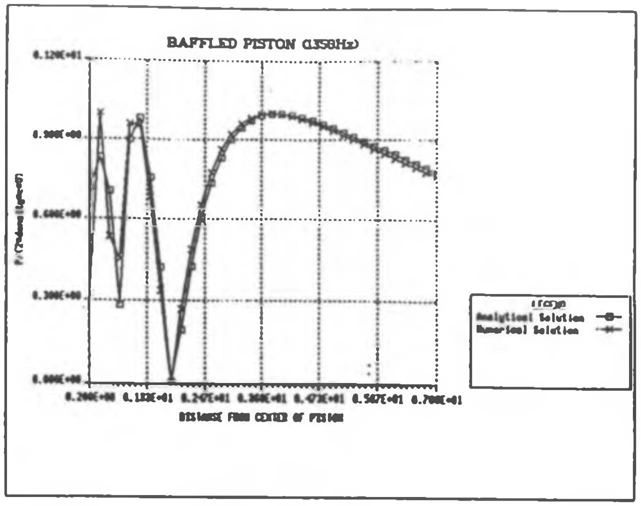

Figure 1. Comparison between analytical and numerical results for the Rayleigh method.

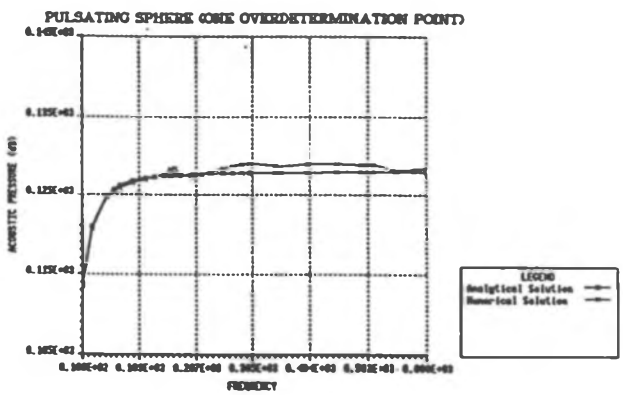

Figure 2. Comparison between analytical and numerical results for the boundary element method (one overdetermination point).

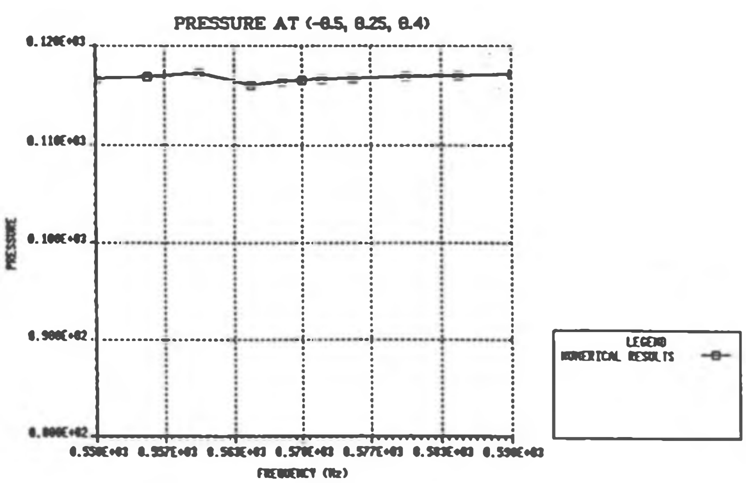

Figure 3. Results Used in the Singularity Study

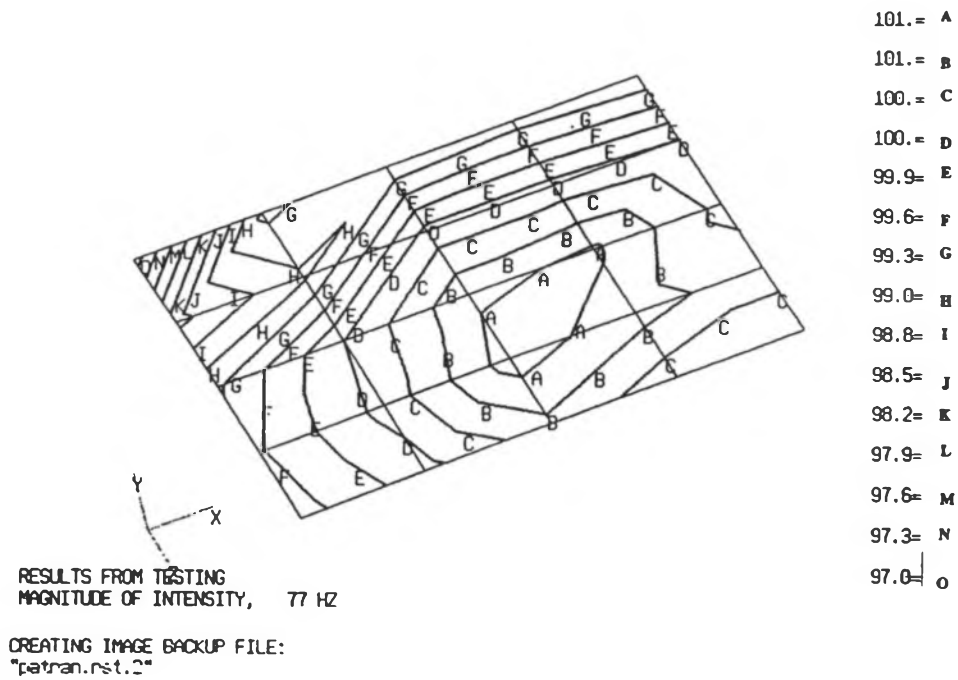

Figure 4.a. Test results for acoustic intensities at field point mesh.

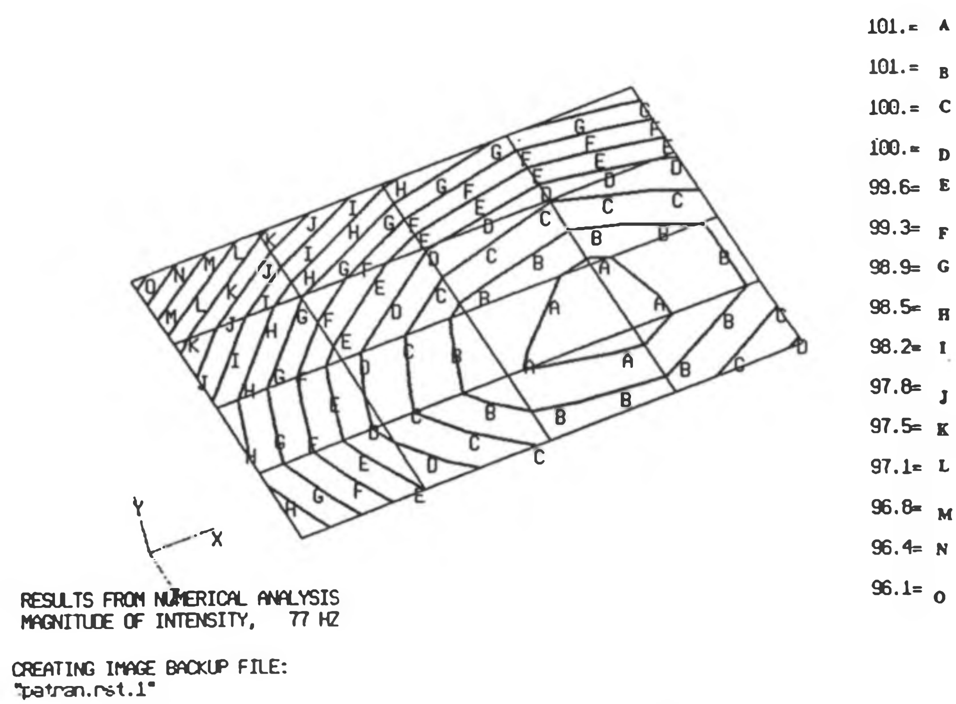

Figure 4.b. Numerical results for acoustic intensities at field point mesh.

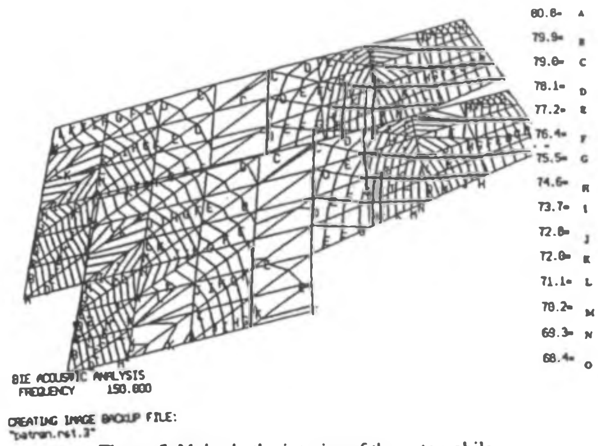

Figure 5. Noise in the interior of the automobile.

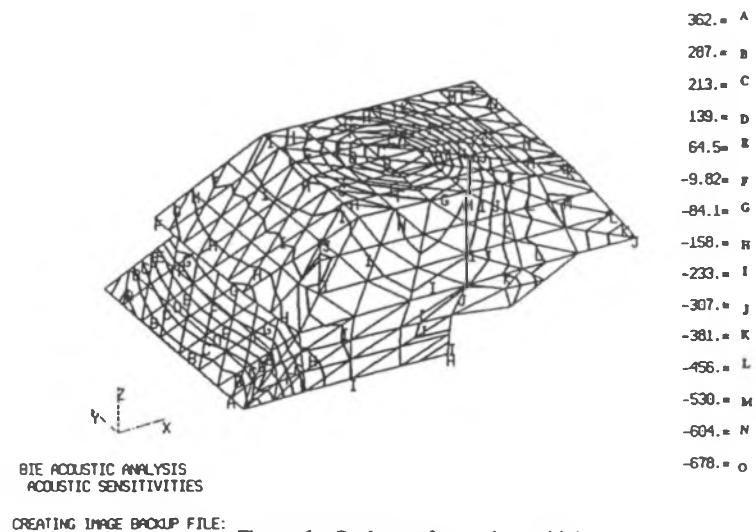

Figure 6 a. Real part of acoustic sensitivity.

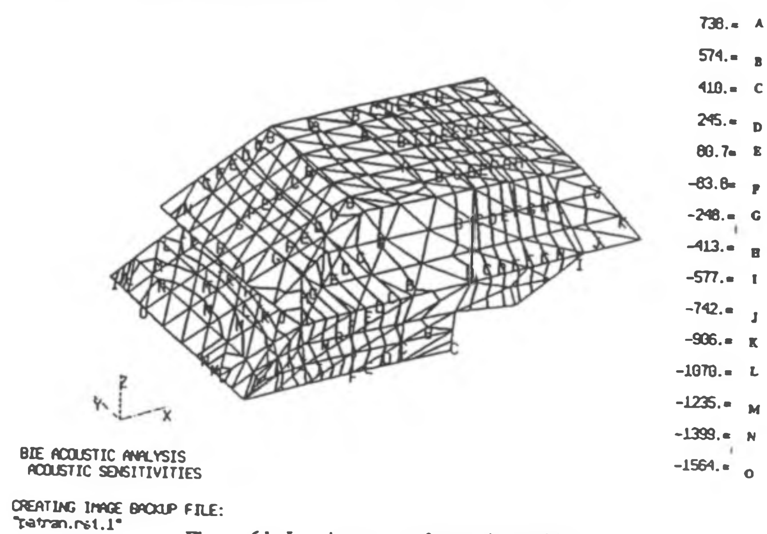

Figure 6.b. Imaginary part of acoustic sensitivity.